IJERMT
International Journal of Engineering Research & Management Technology
ISSN: 2348-4039
March-April-2021 Volume 8, Issue-2
Email: editor@ijermt.org
www.ijermt.org

DIFFERENTIATIONS OF THE HUREWICZ AND MENGER COVERING CHARACTERISTICS IN TOPOLOGICAL SPACES

Dr. P. R. Parihar

Associate Professor

S.P.C. Government College, Ajmer

Abstract:

Throughout the course of this investigation, we will define and discuss covering properties of the semi-Hurewicz type in ditopological texture spaces. In this study, we examine the behavior of semi-Hurewicz and co-semi-Hurewicz selection qualities in the presence of disrepair and differentiations across ditopological texture spaces. Specifically, we look at how these selection qualities behave in the presence of disrepair and differences between texture spaces. An ancient question posed by A.V. Arhangel'skii inquires as to whether or not the Menger property of a Tychonoff space, denoted by X, is maintained by homeomorphisms of the space denoted by Cp(X). Regarding the question of linear homeomorphisms, we give an answer that is positive. In order to achieve this goal, we devise a strategy for researching invariants of linear homeomorphisms of function spaces Cp(X) by observing the positioning of X within its (Cech-Stone) compactification.

keywords: *Hurewicz, Menger, topological*

Introduction

The foundations of the idea of selection principles were laid in the first part of the 19th century. It has been determined that the following is the general form of the classical selection rules that apply in topological spaces:

Let S be an innite set and let A and B be collections of subsets of S. Then the symbol S1(A, B) denes the statement:

For each sequence $(A_n)_{n<\infty}$ of elements of A there is a sequence $(b_n)_{n<\infty}$ such that for each n, we have $b_n \in A_n$, and $\{b_n\}_{n<\infty} \in \mathcal{B}$.

If O denotes the collection of open covers of a topological space (X, T) then the property S1(O, O) , sometimes known as the Rothberger covering property, was developed and first proposed by Rothberger. in [1].

In a similar vein, there is also the selection theory. $\mathsf{S}_{\mathsf{fin}}(\mathcal{A}, \mathcal{B})$ is defined as:

For each sequence $(A_n)_{n<\infty}$ of a progression may be seen in the elements of A. $(B_n)_{n<\infty}$ such that for each n, we have Bn is finite subset of A_n, and $\bigcup_{n<\infty} B_n \in \mathcal{B}$.

K. Menger's contribution to [2] in 1924 was the introduction of a characteristic that is equal to $\mathsf{S}_{\mathsf{fin}}(\mathcal{O}, \mathcal{O})$ as well as was demonstrated by W. Hurewicz in [3] in the year 1925. The piece of property $\mathsf{S}_{\mathsf{fin}}(\mathcal{O}, \mathcal{O})$ is known as the Menger covering property.

In 2016, Sabah et al. demonstrated in [4] that in topological space X, if X satisfies certain conditions, then X possesses the s-Menger property (or the s-Rothberger covering property; see [5]). $\mathsf{S}_{\mathrm{fin}}(s\mathcal{O}, s\mathcal{O})$ (resp. $S_1(s\mathcal{O}, s\mathcal{O})$) where s'$\mathcal{O}$ denotes the family of all semi-open covers of X.

We advise readers who are more interested in the theory of selection principles and its links with other fields of mathematics to look at [6–10] for further information.

Within the scope of this research, we investigate the characteristics of ditopological texture spaces in relation to the following standard Hurewicz property: [3] $\mathsf{U}_{\mathrm{fin}}(\mathcal{A}, \mathcal{B})$: For each sequence $\{A_n\}_{n\in\mathbb{N}}$ of elements of A, which do not contain a finite ub-cover, there exist nite (possibly empty) subsets $B_n \subseteq A_n, n \in \mathbb{N}$ and $\{\bigcup B_n\}_{n\in\mathbb{N}} \in \mathcal{B}$. . It was shown in [11] that the Hurewicz property is of the $\mathsf{S}_{\mathrm{fin}}$-type for appropriate classes of A and B.

Preliminaries

The concept of a texture space was first presented by L. M. Brown in 1992 during a conference on fuzzy systems and artificial intelligence that was held in Trabzon. Brown referred to this concept as fuzzy structure. The first appearance of textures may be traced back to the point-based description of the relationship between the lattices of L-fuzzy sets and the Hutton algebras. This representation offered a productive setting in which to investigate complement free concepts in mathematics. The definition of the texture space is as follows, which we will now recall.

Texture space: [12] If S is a set, a texturing $\Im \subseteq P(S)$ is complete, point separating, completely distributive lattice containing S and ∅, and, for which finite join $\bigvee$ coincides with union $\bigcup$ and arbitrary meet $\bigwedge$ coincides with intersection $\bigcap$. Then the pair $(S, \Im)$ is called the texture space.

A mapping σ :

A mapping $\sigma : \Im \to \Im$ satisfying $\sigma^2(A) = A$, for each $A \in \Im$ and $A \subseteq B$ implies $\sigma(B) \subseteq \sigma(A)$, $\forall\, A, B \in \Im$ is called a complementation on $(S, \Im)$ and $(S, \Im, \sigma)$ is then said to be a complemented texture [12]. The sets $P_s = \bigcap\{A \in \Im \mid s \in A\}$ and $Q_s = \bigvee\{P_t \mid t \in S,\ s \notin P_t\}$ defines conveniently most of the properties of the texture space and are known as *p-sets* and *q-sets* respectively.

For $A \in \Im$ the core A^b of A is defined by $A^b = \{s \in S \mid A \nsubseteq Q_s\}$. The set A^b does not necessarily belong to $\Im$.

If $(S, P(s))$, $(\mathcal{L}, \Im_2)$ are textures, then the product texture of $(S, P(S))$ and $(\mathcal{L}, \Im_2)$ is $P(S) \otimes \Im_2$ for which $\overline{P}_{(s,t)}$ and $\overline{Q}_{(s,t)}$ denotes the *p-sets* and *q-sets* respectively. For $s \in S, t \in \mathcal{L}$ we have p-sets and q-sets in the product space as following :

$$\overline{P}_{(s,t)} = \{s\} \times P_t$$

$$\overline{Q}_{(s,t)} = (S \backslash \{s\} \times T) \cup (S \times Q_t).$$

Direlation: [13] Let $(S, \Im_1), (\mathcal{L}, \Im_2)$ be textures. Then for $r \in \mathcal{P}(S) \otimes \Im_2$ satisfying:

(R_1) $r \not\subseteq \overline{Q}_{(s,t)}$ and $P_{\acute{s}} \not\subseteq Q_s$ implies $r \not\subseteq \overline{Q}_{(\acute{s},t)}$,
(R_2) $r \not\subseteq \overline{Q}_{(s,t)}$ then there is $\acute{s} \in S$ such that $P_s \not\subseteq Q_{\acute{s}}$ and $r \not\subseteq \overline{Q}_{(\acute{s},t)}$,
is called *relation* and for $R \in \mathcal{P}(S) \otimes \Im_2$ such that:
(CR_1) $\overline{P}_{(s,t)} \not\subseteq R$ and $P_s \not\subseteq Q_{\acute{s}}$ implies $\overline{P}_{(\acute{s},t)} \not\subseteq R$,
(CR_2) If $\overline{P}_{(s,t)} \not\subseteq R$ then there exists $\acute{s} \in S$ such that $P_{\acute{s}} \not\subseteq Q_s$ and $\overline{P}_{(\acute{s},t)} \not\subseteq R$,
is called a *corelation* from $(S, P(S))$ to $(\mathcal{L}, \Im_2)$. The pair (r, R) together is a direlation from $(S, \Im_1)$ to $(\mathcal{L}, \Im_2)$.

Lemma 2.1 ([13]) Let (r, R) be a dereliction from $(S, \Im_1)$ *to* $(T, \Im_2)$, J J be an index set $A_j \in \Im_1, \forall j \in J$ *and* $B_j \in \Im_2, \forall j \in J$. Then:

$$(1)\ r^{\leftarrow}\Big(\bigcap_{j\in J} B_j\Big) = \bigcap_{j\in J} r^{\leftarrow} B_j \text{ and } R^{\rightarrow}\Big(\bigcap_{j\in J} A_j\Big) = \bigcap_{j\in J} R^{\rightarrow} A_j,$$
$$(2)\ r^{\rightarrow}\Big(\bigvee_{j\in J} A_j\Big) = \bigvee_{j\in J} r^{\rightarrow} A_j \text{ and } R^{\leftarrow}\Big(\bigvee_{j\in J} B_j\Big) = \bigvee_{j\in J} R^{\leftarrow} B_j.$$

Difunction: Let be a direlation from $(S, \Im_1)$ to $(\mathcal{L}, \Im_2)$. Then $(f, F) : (S, \Im_1) \to (\mathcal{L}, \Im_2)$ is a malfunction in the event that it satisfies both of the following conditions:

(DF1) For $s, \acute{s} \in S, P_s \not\subseteq Q_{\acute{s}} \Longrightarrow t \in \mathcal{L}$ with $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(\acute{s},t)} \not\subseteq F$.

(DF2) For $t, t' \in \mathcal{L}$ and $s \in S$, $f \not\subseteq Q_{(s,t)}$ and $\overline{P}_{(s,t')} \not\subseteq F \Longrightarrow P_{t'} \not\subseteq Q_t$.

Dentition 2.2([13]). Let $(f, F) : (S, \Im_1) \to (\mathcal{L}, \Im_2)$ be a dysfunction. For A ∈ =1, the image f −→(A) and co image F −→(A) are defined as:

$$f^{\rightarrow}(A) = \bigcap\{Q_t : \forall s,\ f \not\subseteq \overline{Q}_{(s,t)} \Longrightarrow A \subseteq Q_s\},$$

$$F^{\rightarrow}(A) = \bigvee\{P_t : \forall s,\ \overline{P}_{(s,t)} \not\subseteq F \Longrightarrow P_s \subseteq A\},$$

and for B $\in \Im_2$, the inverse image f ←−(B) and inverse coimage F ←−(B) are dened as:

$$f^{\leftarrow}(B) = \bigvee\{P_s : \forall t,\ f \not\subseteq \overline{Q}_{(s,t)} \Longrightarrow P_t \subseteq B\},$$

$$F^{\leftarrow}(B) = \bigcap\{Q_s : \forall t,\ \overline{P}_{(s,t)} \not\subseteq F \Longrightarrow B \subseteq Q_t\}.$$

The inverse image and the inverse co image of a difunction are equal, but the image and the coimage are often not equal to one another.

Lemma 2.3([13]). For a direlation (f, F) *from* $(S, \Im_1)$ *to* $(T, \Im_2)$ the following are equivalent:

(1) (f , F) is a direlation.

(2) The following inclusion holds:

(a) $f^{\leftarrow}(F^{\rightarrow}(A)) \subseteq A \subseteq F^{\leftarrow}(f^{\rightarrow}(A)); \forall A \in \Im_1$, *and*
(b) $f^{\rightarrow}(F^{\leftarrow}(B)) \subseteq B \subseteq F^{\rightarrow}(f^{\leftarrow}(B)); \forall B \in \Im_2$
(3) $f^{\leftarrow}(B) = F^{\leftarrow}(B); \forall B \in \Im_2$.

Definition 2.4 ([13]). *Let $(f, F) : (S, \Im_1) \to (\mathcal{L}, \Im_2)$ be a difunction. Then (f, F) is called surjective if it satisfies the condition:*
(SUR) For $t, t' \in \mathcal{L}, P_t \nsubseteq Q_{t'} \Longrightarrow \exists s \in S, f \nsubseteq Q_{(s,t')}$ and $\overline{P}_{(s,t)} \nsubseteq F$.
Similarly, (f, F) is called injective if it satisfies the condition
(INJ) For $s, \acute{s} \in S$, and $t \in \mathcal{L}$ with $f \nsubseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(\acute{s},t)} \nsubseteq F \Longrightarrow P_s \nsubseteq Q_{\acute{s}}$.

We now recall the notion of ditopology on texture spaces.

Definition 2.5 ([14]). *A pair (τ, κ) of subsets of $\Im$ is said to be a ditopology on a texture space $(S, \Im)$, if $\tau \subseteq \Im$ satisfies:*
(1) $S, \emptyset \in \tau$.
(2) $G_1, G_2 \in \tau$ implies $G_1 \cap G_2 \in \tau$ and
(3) $G_\alpha \in \tau, \alpha \in I$ implies $\bigvee_\alpha G_\alpha \in \tau$,
and $\kappa \subseteq \Im$ satisfies:
(1) $S, \emptyset \in \kappa$.
(2) $F_1, F_2 \in \kappa$ implies $F_1 \cup F_2 \in \kappa$ and
(3) $F_\alpha \in \kappa, \alpha \in I$ implies $\bigcap F_\alpha \in \kappa$,

Both the inverse image and the inverse co image of a difunction have the same value; however, the image and the coimage of a difunction do not always have the same value.

Note that in ditopology, in general, we do not consider there to be any link between the open sets and the closed sets. In the event that the texture space is supplemented $(S, \Im, \sigma)$, τ and κ are connected by the relation κ = σ(τ), where σ is a complementation on (S, =), that is an inclusion reversing involution $\sigma : \Im \to \Im$, then we calcomplemented ditopology on $(S, \Im)$. A complemented ditopological texture space is denoted by $(S, \Im, \sigma, \tau, \kappa)$. In this specific instance, we have $\sigma(\overline{A}) = (\sigma(A))^\circ$ and $\sigma(A^\circ) = \overline{(\sigma(A))}$, where $(\,)^\circ$ both the interior and the closing are denoted by this symbol. Keep in mind that this pertains to a ditopology. (τ, κ) on $(S, \Im)$, for $A \in \Im$ the closure of A for the ditopology (τ, κ) is denoted by (A) and defined by

$$(\overline{A}) = \bigcap\{F \in \kappa : A \subseteq F\},$$

The interior of A is represented by the symbol (A), and it is defined as

$$(A)^\circ = \bigvee\{G \in \tau : G \subseteq A\}$$

The reader is directed to visit for definitions of any terminology that are not included here.[6, 13, 15].

In 1963, Norman Levine published [16] where he presented for the first time the concept of semi-open sets in topological spaces. This notion of semi-open sets was first introduced in topological spaces by Dost in 2012. In 2012, he expanded it to ditopological texture spaces. [17].

It is known from [17] that in a ditopological texture space $(S, \Im, \tau, \kappa)$:

1. $A \in \Im$ is semi-open if and only if there exists a set $G \in O(S)$ such that $G \subseteq A \subseteq \overline{G}$.
2. $B \in \Im$ is semi-closed if and only if there exists a set $F \in C(S)$ such that $(F)^{\circ} \subseteq B \subseteq F$.
3. $O(S) \subseteq SO(S)$ and $C(S) \subseteq SC(S)$. The collection of all semi-open (resp. semi-closed) sets in $\Im$ is denoted by $SO(S, \Im, \tau, \kappa)$ or simply $SO(S)$ (resp. $SC(S, \Im, \tau, \kappa)$ or simply $SC(S)$). $SR(S)$ is the collection of all the semi-regular sets in S. A set A is semi-regular if A is semi-open as well as semi-closed in S.
4. Arbitrary join of semi-open sets is semi-open.
5. Arbitrary intersection of semi-closed sets is semi-closed.

If the ditopological texture space (S, =,,) has a semi-open representation of A, then its complement might not be semi-closed. Every semi-open set is also open, however a set that is just semi-open could not be open. It's possible that the intersection of two semi-open sets won't be semi-open, but the intersection of an open set and a semi-open set is certain to be semi-open. However, in the situation of complementing ditopological texture space (S, =,,,), A = is semi-open if and only if (A) is semi-closed. In general, there is no link between semi-open and semi-closed sets. In this notation, () indicates the semi-interior, while () indicates the semi-closure.

Conclusion

In this study, we explore the behavior of semi-Hurewicz and co-semi-Hurewicz selection qualities in the presence of disrepair and differentiations between ditopological texture spaces. Specifically, we look at how these selection qualities behave when disrepair and differentiations occur. A Hurewicz space is a topological space in mathematics that fulfills a certain basic selection principle that generalizes -compactness. This principle is known as the Hurewicz selection principle. A space is said to be a Hurewicz space if, for any series of open covers of the space, there are finite sets in such a way that every point in the space belongs to all but a finitely number of the sets.

References

[1] Rothberger, F., Eine Verscharfungder EigenschaftC, Fund. Math., 1938, 30, 50-55
[2] Menger, K., Einige Überdeckungssätze der Punktmengenlehre, Sitzungsberischte Abt. 2a, Mathematik, Astronomie, Physik, Meteorologie und Mechanik (Wiener Akademie, Wien), 1924, 133, 421-444
[3] Hurewicz, W., Über eine Verallgemeinerung des Borelschen Theorems, Math. Z., 1925, 24, 401-421
[4] Sabah, A., Khan, M., Kočinac, Lj. D. R., Covering properties dened by semi-open sets, J. Nonlinear Sci. Appl., 2016, 9, 4388-4398
[5] Kočinac, Lj. D. R., Sabah, A., Khan, M., Seba, D., Semi-Hurewicz spaces, Hacet. J. Math. Stat., 2017, 46(1), 53-66
[6] Kočinac, Lj. D. R., Selected results on selection principles, In Proceedings of Third Seminar on Geometry and Topology, (Tabriz, Iran), 2004, July 15-17, Tabriz, Iran, 71-104
[7] Sakai, M., Scheepers, M., The combinatorics of open covers, In: K.P. Hart, J. van Mill, P. Simon (Eds.), Recent Progress in General Topology III, Atlantis Press, 2014, 751-799
[8] Scheepers, M., The combinatorics of open covers I: Ramsey theory, Topology Appl., 1996, 69, 31-62
[9] Scheepers, M., Selection principles and covering properties in Topology, Note Mat., 2003, 22(2), 3-41
[10] Tsaban, B., Some new directions in innite-combinatroial topology, in: J. Bagaria, S. Todorčevič (Eds.), Set Theory, in: Trends in Mathematics, Birkhauser, 2006, 225-255
[11] Kočinac, Lj. D. R., Scheepers, M., Combinatorics of open covers (VII): Groupability, Fund Math., 2003, 179, 131-155
[12] Brown, L. M., Diker, M., Ditopological texture spaces and intuitionistic sets, Fuzzy Sets Syst., 1998, 98, 217-224
[13] Brown, L. M., Ertürk, R., Dost, S., Ditopological texture spaces and fuzzy topology,I. Basic concepts, Fuzzy Sets Syst., 2004, 147(2), 171-199

[14] Kočinac, Lj. D. R., özçağ, S., Selection properties of texture structures, Topology Appl., 2015, 192, 198-207
[15] Brown, L. M., Gohar, M. M., Compactness in ditopological texture spaces, Hacettepe J. Math. Stat., 2009, 38, 21-43
[16] Levine, N., Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 1963, 70, 36-41
[17] Dost, Ş., Semi-open and semi-closed sets in ditopological texture spaces, J. Adv. Math. Stud., 2012, 5(1), 97-110